# Nomographs for Parabolic Reflector Antennas

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## Introduction

Reflector antennas are the primary high-performance antennas of choice, especially when high gain, high *G/T* ratio and minimal back- and side-lobe artifacts are required. Simple functionality and inherent performance superiority contribute to widespread use of the reflector antennas for both terrestrial and space communications. To better utilize reflector-based antennas and to understand their principles, I have developed and plotted nomographs that help to design and study their behaviours.

## Fig 1. Gain of a Parabolic Antenna

Gain of a Parabolic Antenna is calculated using the following formula:

$$\boldsymbol{G = 10\log\left(\eta\frac{\pi^2D^2}{\lambda^2}\right)} \qquad [\text{dBi}] \qquad (1)$$

where $G$ is the antenna gain,
$D$ is the reflector diameter,
$\lambda$ is the wavelength (S - band 125mm, X - band 28.6mm), and
$\eta$ is the total efficiency.

The efficiency consists of many individual components [1,2]; consequently, the total efficiency is a function of their product.

There are four main efficiency components:

1. the illumination efficiency,
2. the spillover efficiency,
3. the phase efficiency, and
4. the crosspolarization efficiency.

The efficiency of electrically small parabolic dish antennas is substantially degraded by various size-specific factors, such as the large size of the feed in comparison to a relatively small reflector, etc. Furthermore, it is usually not possible to substantially reduce the feed's size. More can be found in [3].

## Fig 2. Feed Subtended Angle – Prime Focus Configuration

Feed Subtended Angle is the angle in which the feed sees rims of a reflector from the focal point. It is calculated based using the following formula:

$$\boldsymbol{SA = 4\arctan\frac{1}{4fD}} \qquad [\text{deg.}] \qquad (2)$$

where

*SA* is the total feed subtended angle,
*f* is the focal length and
*D* is the reflector diameter.

### Fig 3. Feed Subtended Angle – Offset-fed Reflector

The basic offset-fed reflector geometry is displayed in the figure below.

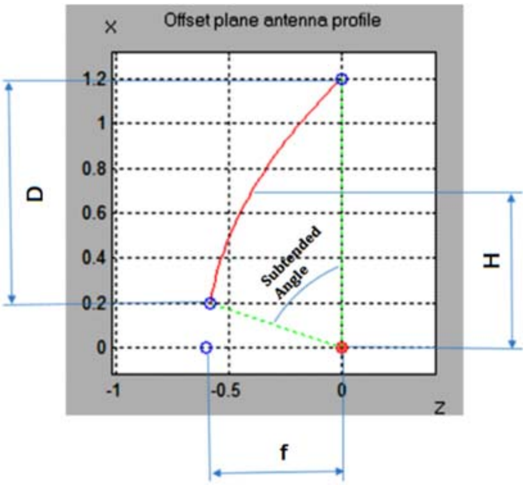


**Fig. 3.1** The basic offset-fed reflector geometry.

The offset height *H* is added to the variables listed above.

The subtended angle is calculated using the following formula:

$$SA = 2\arctan\frac{8fD}{16f^2 + 4H^2 - D^2} \quad [\text{deg}] \tag{3}$$

In Fig. 3, the subtended angle is shown in multiples of the reflector diameter. The prime focus configuration (black line) is acquired when the offset height $H = 0$, and its curve is identical with curve plotted in Fig. 2.

**Note**, that the offset reflectors are almost always designed with offset height $H > 0.5\ D$, to use their main advantage not to block the reflector by a feed.

If the parameters of our reflector are unknown, we can get them by measuring the reflector size. To calculate desired values, various free calculators or software can be used. [4,5]

## Fig. 4 Added Edge Taper

Due to a parabolic shape of the reflector and the spherical spread, field emanating from a feed is reaching dish vertex with higher intensity than the rim of the reflector. As lower *f/D* ratio is as higher is spherical spread loss. The added edge taper for the prime focus reflector is calculated the following formula:

$$\boldsymbol{AET = 20\log\cos^2\left(\arctan\frac{1}{4fD}\right)} \quad [\mathrm{dB}] \tag{4}$$

where

*AET* is the added edge taper,
*f* is the focal length, and
*D* is the reflector diameter.

**Note,** that for an offset-fed reflector, *AET* differs for the upper and lover rims, as the offset-fed reflectors have asymmetric structure. Nevertheless, its dispersion is relatively low thanks to the high *f/D* ratio.

## Fig. 5 Feed Gain vs Parabolic Reflector *f/D* Ratio

Requrements of well performing feeds are according to [6] following:

- The ideal feed produces uniform amplitude and phase distribution which compensates the spherical spreading loss and does not have spillover (this case cannot be realized in practice).
- The feed pattern should be rotationally symmetric (balanced feed).
- The feed pattern should be such that the reflector edge illumination is about 10-11 dB (for the best gain), or 13-15 dB (for the best *G/Ta* ratio).
- The feed should have a point phase center and the phase center should be positioned at the focal point of the reflector.
- The feed should be small in order to reduce the reflector blockage.
- The feed should have low cross polarization, usually below 30 dB.
- The above characteristics should hold over the desired operational frequency band.

We can see that some of these requirements are contradictory. With some simplifications (symmetrical pattern, constant phase, 100% efficiency i.e. directivity = gain, prime focus configuration), the feed pattern can be approximated by the function

$$U(\theta) = cos^{2N}\theta \quad \text{for } \theta \leq 90^o \tag{5.0}$$

Then for the added edge taper -10 dB

$$\boldsymbol{G_f = 10\log\left(\frac{-2}{\log\left[\cos\left(\frac{SA}{2}\right)\right]} + 2\right)} \quad [\mathrm{dBi}] \tag{5}$$

Where

$G_f$ is the feed gain for the particular subtended angle, and

$SA$ is the feed total subtended angle.

For better orientation, a diagram is plotted for *f/D*, corresponding to the particular *SA*.

**Note, that this approach has limited accuracy and can be used as an approximation of the feed requirements for a particular reflector. For the final antenna design, feed's actual radiation pattern should be used.**

### Fig 6. Helical Antenna

The helical antenna can be used as stand-alone or as a feed for parabolic reflector antenna. These technically elegant antennas were invented by John D. Kraus [7].

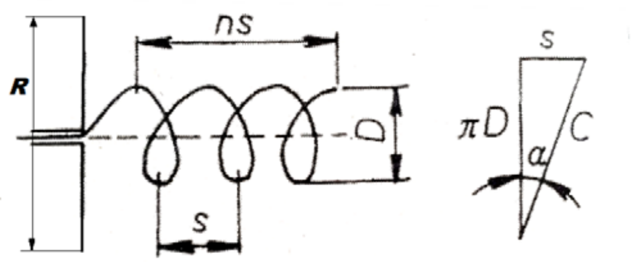


Fig. 6.1 The geometry of helical antenna.

The main mechanical dimensions were proposed in [7]. In contrast with the book, I enlarged ground plane of helical antenna from 0.75λ to 1λ, see the Fig 6.1. Thanks to Czech Technical University in Prague, I was able to create a model and to calculate the helical antenna performance using the CST MW STUDIO 2020 software. Antenna parameters are following:

$C =$ circumference of the helix = 1λ = 125 mm,
$D$ = diameter of the helix (center to center) = 39.79 mm,
$s =$ spacing between the turns (center to center) = 28.86 mm,
$\alpha =$ the pitch angle = $13^0$,
$n =$ numbers of turns,
$R =$ ground plane diameter (reflector) = 125 mm.

I choose wire diameter to be 4 mm.

**Example 1.**

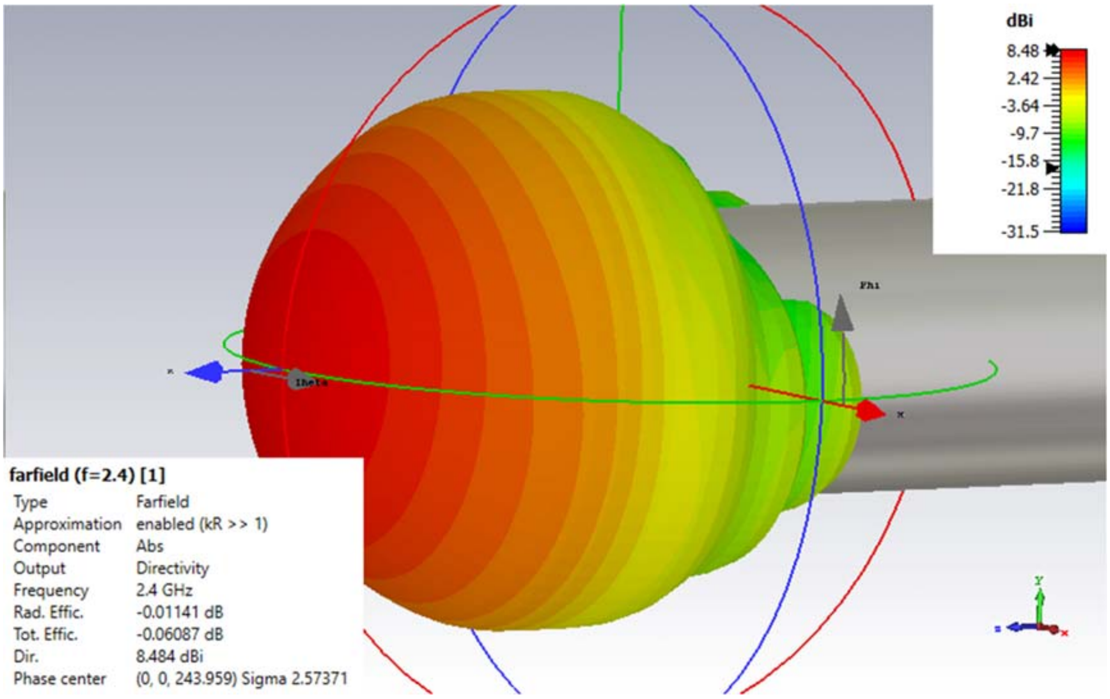
Find the optimal diameter of the circular waveguide feed (coffee can feed) for the prime focus reflector (Primesat) with *f/D* = 0.41. The operating wavelenght is $\lambda = 125$ mm. Operating range for pure basic mode TE11 in circular waveguide is given by following formulaes:

$$D_{wmin} = \frac{1.841\lambda}{\pi} \leq D_w \leq \frac{2.405\lambda}{\pi} = D_{wmax} \qquad (7)$$

where

$D_w$ is the inner diameter of the waveguige.

Using the Equation 7, we get $D_{wmin}$= 73.25 mm, and $D_{wmax}$ = 95.69 mm, while the tubes with inner diameter from 75, 80, 85, 90 up to 95mm are available in hardware shops. Using CST MW Studio, I calculated gains for these available tubes, for the calculated feed performance, se the Table below.

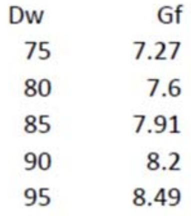


**farfield (f=2.4) [1]**

| | |
|---|---|
| Type | Farfield |
| Approximation | enabled (kR >> 1) |
| Component | Abs |
| Output | Directivity |
| Frequency | 2.4 GHz |
| Rad. Effic. | -0.01141 dB |
| Tot. Effic. | -0.06087 dB |
| Dir. | 8.484 dBi |
| Phase center | (0, 0, 243.959) Sigma 2.57371 |

Fig.7.1 The radiation pattern for the circular waveguide feed with the diameter of 95 mm.

Tab. 7.2



| Dw | Gf |
|---|---|
| 75 | 7.27 |
| 80 | 7.6 |
| 85 | 7.91 |
| 90 | 8.2 |
| 95 | 8.49 |

Based on the Figure 5, we can determine the most suitable size of the feed for our reflector, which is a circular waveguide with the inner diameter of **95 mm**.

### Example 2.

For the offset reflector with *f/D* = 0.67, *SA* = 70 deg. (Gibertini, model OP 75 LN), consider a helical feed. Determine how many turns of helical feeds is optimal for this reflector.

Step 1

Since the offset reflector has an assymetric structure, it is common technique to find the feed for the prime focus configuration with the same subtended angle *SA*, then determine feed gain for this subsidiary reflector. Based on Figure 2 and Figure 3, we can estimate that for *SA* = 70 deg, the *f/D* is equal to approximatelly 0.78 of the subsidiary prime-focus reflector.

Step 2

Based on the Fig. 5, we can estimate that for *f/D* ratio of 0.78, we need feed with gain of about 13.7 dBi.

Step 3

Based on the Figure 6, we can estimate, that to achieve gain 13.7 dBi, we need a helical antenna with about **10 turns**.

**Remarks**

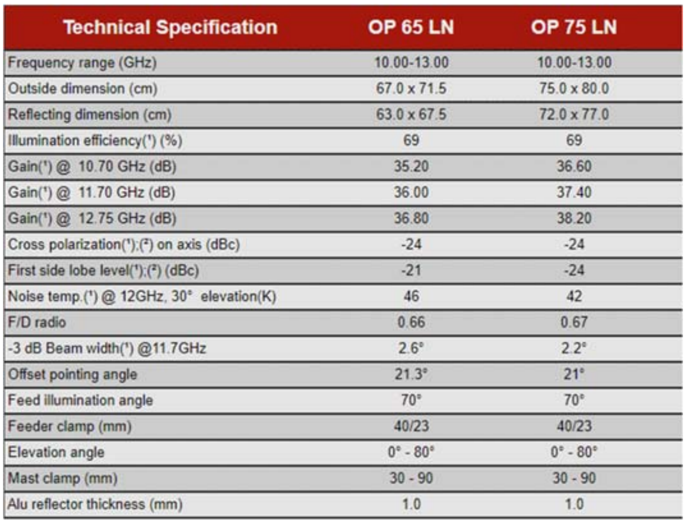
The antenna assembly from the Example 2 achieves a gain of about 23.5 dBi (see Figure 1). Lets assume, that we are using Tx power of 1.5 W for the satellite operatios. Our EIRP is then about 336 W. The same EIRP can be achieved with the compact helical antenna (which must have an opposite polarization) and the power of 15 W. It is questionable, whether it is better to use a stand-alone simple compact helical antenna with a higher power, which is very easy to procure, or relatively large 75 cm dish assembly instead.

### Example 3.

For the given reflector from Example 2, (Gibertini, model OP 75 LN), determine how many turns of helical feeds is optimal. Use the full-wave analysis method (real radiation pattern).

Step 1

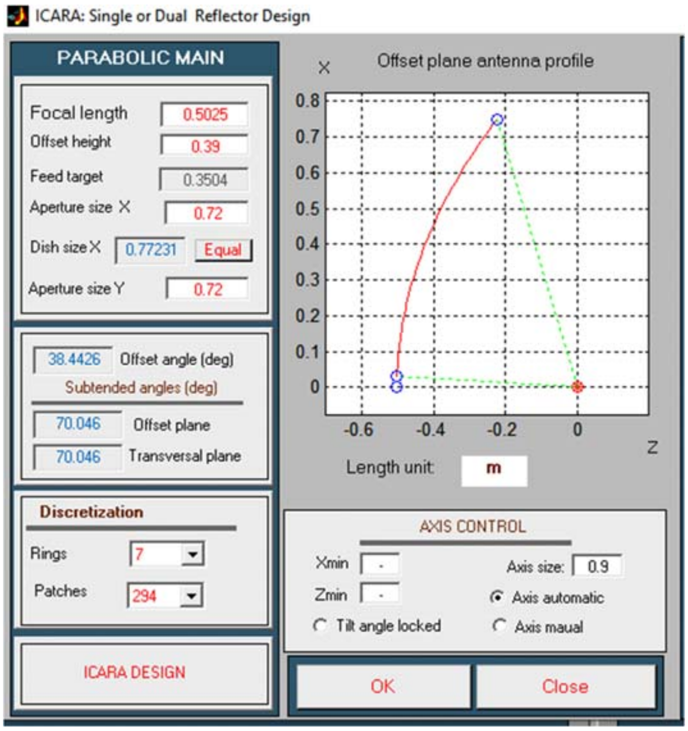
To construct the given reflector in CST MW Studio software, we must find reflector geometry. We can find basic informations on the producer website, see Fig 7.2



| Technical Specification | OP 65 LN | OP 75 LN |
|---|---|---|
| Frequency range (GHz) | 10.00-13.00 | 10.00-13.00 |
| Outside dimension (cm) | 67.0 x 71.5 | 75.0 x 80.0 |
| Reflecting dimension (cm) | 63.0 x 67.5 | 72.0 x 77.0 |
| Illumination efficiency(1) (%) | 69 | 69 |
| Gain(1) @ 10.70 GHz (dB) | 35.20 | 36.60 |
| Gain(1) @ 11.70 GHz (dB) | 36.00 | 37.40 |
| Gain(1) @ 12.75 GHz (dB) | 36.80 | 38.20 |
| Cross polarization(1);(2) on axis (dBc) | -24 | -24 |
| First side lobe level(1);(2) (dBc) | -21 | -24 |
| Noise temp.(1) @ 12GHz, 30° elevation(K) | 46 | 42 |
| F/D radio | 0.66 | 0.67 |
| -3 dB Beam width(1) @11.7GHz | 2.6° | 2.2° |
| Offset pointing angle | 21.3° | 21° |
| Feed illumination angle | 70° | 70° |
| Feeder clamp (mm) | 40/23 | 40/23 |
| Elevation angle | 0° - 80° | 0° - 80° |
| Mast clamp (mm) | 30 - 90 | 30 - 90 |
| Alu reflector thickness (mm) | 1.0 | 1.0 |

**Fig. 7.2** Technical specification of the parabolic reflector.

To find the offset height *H,* ICARA software was applied [4]. See Fig 7.3.



ICARA: Single or Dual Reflector Design

PARABOLIC MAIN

Focal length 0.5025

Offset height 0.39

Feed target 0.3504

Aperture size X 0.72

Dish size X 0.77231

Aperture size Y 0.72

38.4426 Offset angle (deg)

Subtended angles (deg)

70.046 Offset plane

70.046 Transversal plane

Discretization

Rings 7

Patches 294

Length unit m

AXIS CONTROL

Xmin -

Zmin -

Axis size: 0.9

● Axis automatic

○ Axis maual

○ Tilt angle locked

**Fig. 7.3** The reflector geometry in ICARA software

Reflector parameters are:

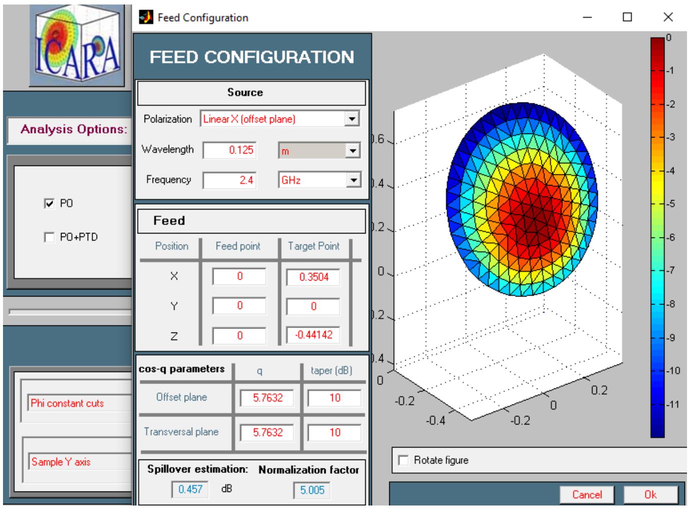
$D = 720$ mm, $f = 502.5$ mm, $H = 390$, $SA = 70$ deg.

From [1] , feed angle $\psi_f = 2 \arctan \frac{H}{2f} = 42.418$ deg

Dimensions of the basic paraboloid, from the offset reflector is excluded, are

$D_b = 2(H + D/2) = 1500$ mm, $f = 502.5$ mm, $f/D_b = 0.335$

Once we have the ICARA software open, we can also check the feed Gain calculation in Example 2. See Fig. 7.4



Feed Configuration

FEED CONFIGURATION

Analysis Options:

☑ PO

☐ PO+PTD

Phi constant cuts

Sample Y axis

Source

Polarization: Linear X (offset plane)

Wavelength: 0.125 m

Frequency: 2.4 GHz

Feed

| Position | Feed point | Target Point |
|---|---|---|
| X | 0 | 0.3504 |
| Y | 0 | 0 |
| Z | 0 | -0.44142 |

| cos-q parameters | q | taper (dB) |
|---|---|---|
| Offset plane | 5.7632 | 10 |
| Transversal plane | 5.7632 | 10 |

Spillover estimation: 0.457 dB

Normalization factor: 5.005

☐ Rotate figure

**Fig. 7.4**. Feed configuration in ICARA software. The software can use directivity (Gain) approximation of the feed radiation pattern by using formula (5.0), $U(\theta) = cos^{2N}\theta$. The exponent $N$ is expressed as the letter $q$ in this program.

From Milligan [1]

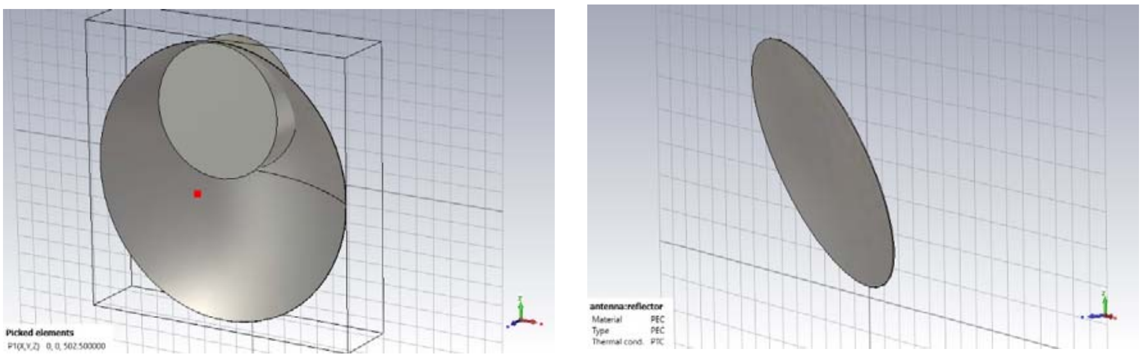
$$\text{directivity } (G_f) = 2(2N + 1) \;\; \text{(ratio)} \tag{8}$$

for $N = 5.7632$ is $G_f = 13.98$ dBi

We calculated $G_f = 13.7$ dBi in Example 2, which is very good agreement.

Step 2

We can construct parabolic reflector in CST MW Studio now. See Fig. 7.5.



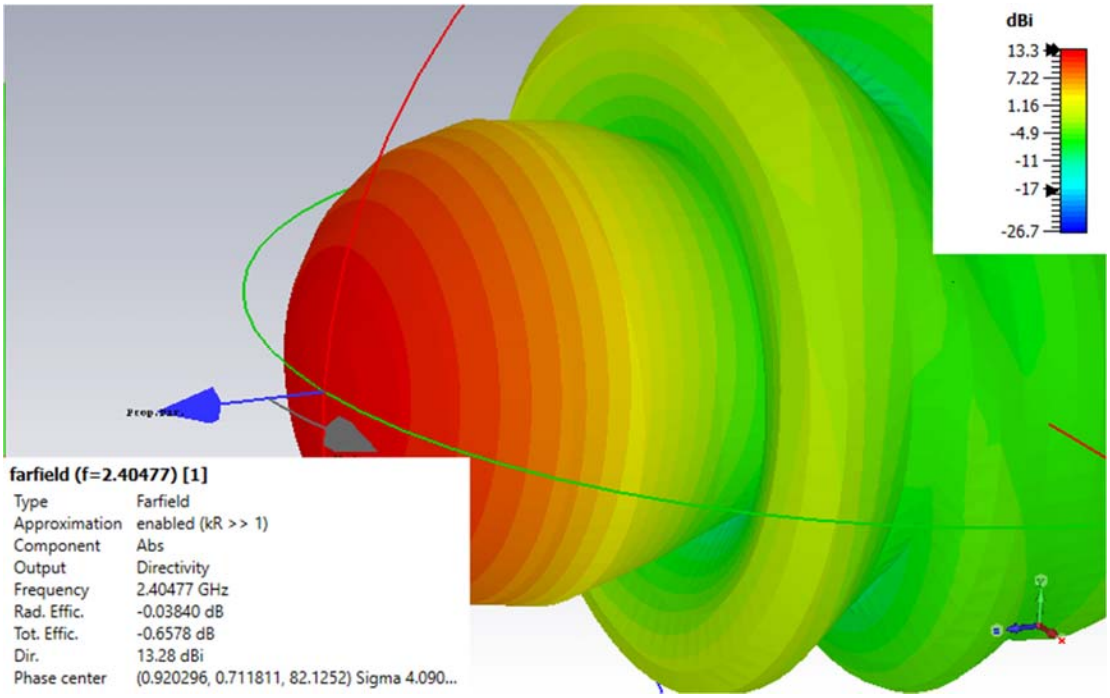
Picked elements
P1(X,Y,Z) 0, 0, 502.500000

antenna:reflector
Material PEC
Type PEC
Thermal cond. PTC

**Fig. 7.5** The offset reflector construction in CST MW Studio software.

Step 3.

I performed calculation of the helical antenna with 5, 6, 7, 8, 9 and 10 turns. T-solver was applied for these calculations. See Figures 7.6 and 7.8 Six farfield sources were produced as the result.



**farfield (f=2.40477) [1]**

| | |
|---|---|
| Type | Farfield |
| Approximation | enabled (kR >> 1) |
| Component | Abs |
| Output | Directivity |
| Frequency | 2.40477 GHz |
| Rad. Effic. | -0.03840 dB |
| Tot. Effic. | -0.6578 dB |
| Dir. | 13.28 dBi |
| Phase center | (0.920296, 0.711811, 82.1252) Sigma 4.090... |

**Fig. 7.6** 3D radiation patter of the helical antenna with 9 turns

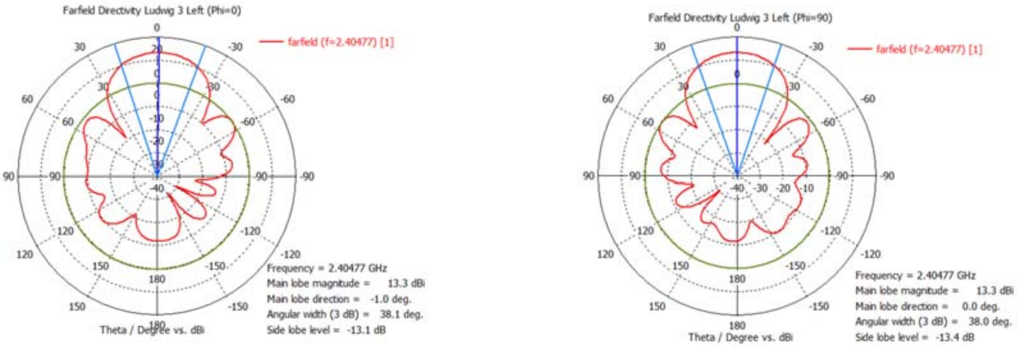
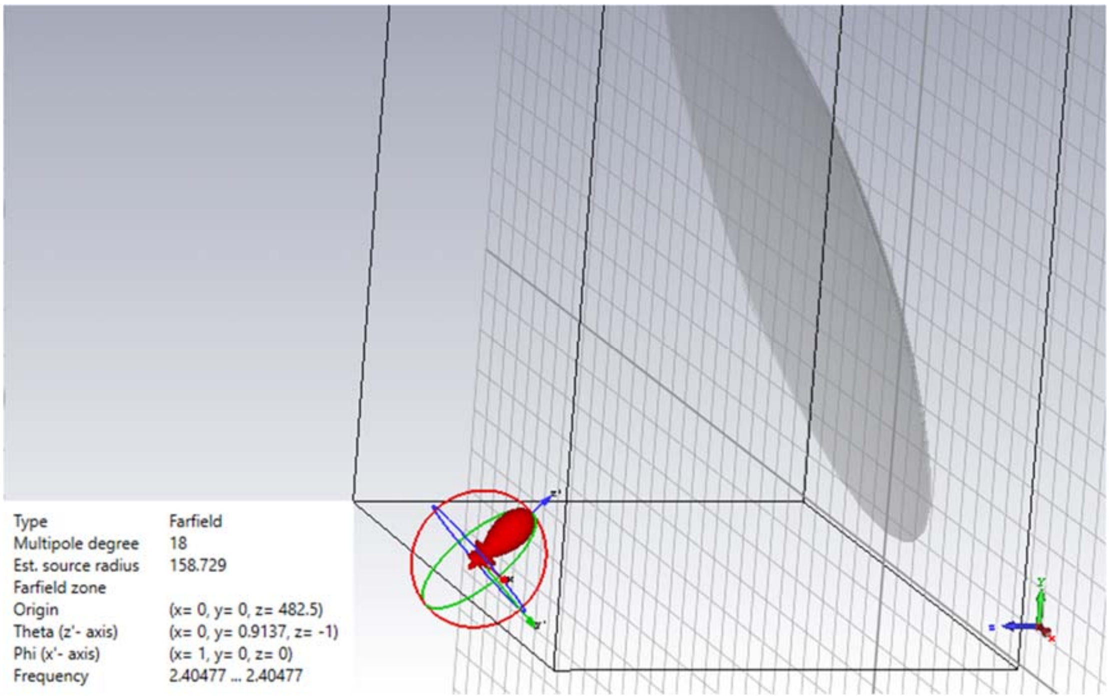


**Fig. 7.7** Polar plot of the radiation pattern of the helical antenna with 9 turns. Phi = 0, left, Phi = 90, right.

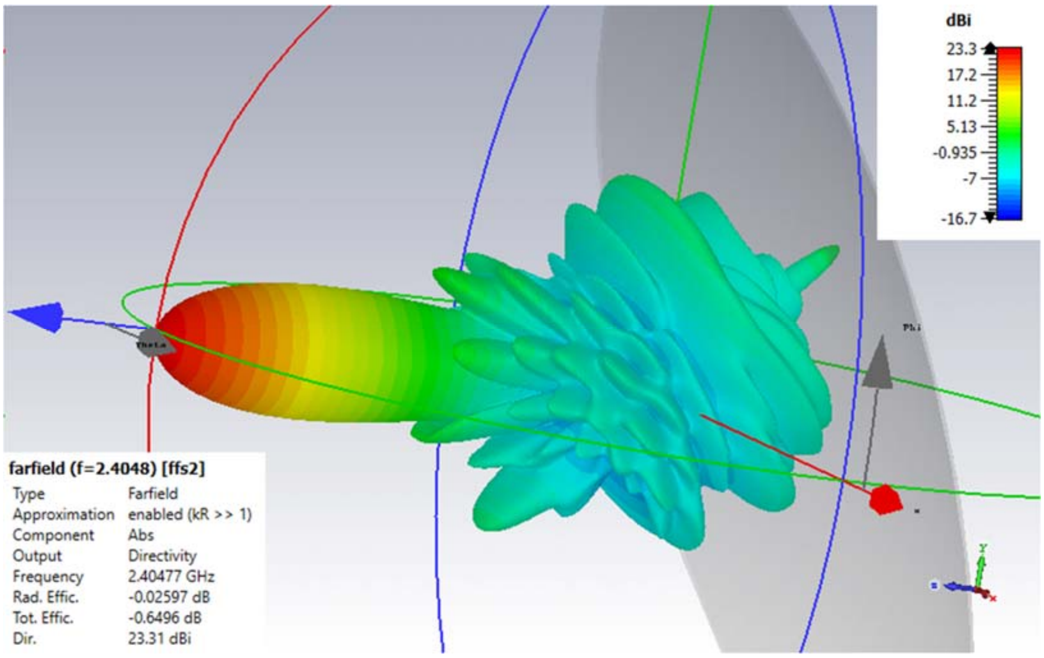
Step 4

Since dish antenna assembly is very large structure, I-solver was applied for folliwing calculations. I used precalculated farfield sources to get entire performance of the dish assembly. See Fig. 7.8. However, some discrepancies between calculated phase center and the best feed position (for max gain) were investigated. The optimal phase center differed from -20 up to 30 mm from calculated position. To eliminate this discrepancy, I created the antenna model with auxiliary prime focus parabolic reflector (less variables) having the same diameter and subtended angle as the Gibertini reflector. Using this auxiliary assembly, the best feed position for each helical antenna was found. Thus modified phase center I used for final evaluation. See Fig. 7.9.



| | |
|---|---|
| Type | Farfield |
| Multipole degree | 18 |
| Est. source radius | 158.729 |
| Farfield zone | |
| Origin | (x= 0, y= 0, z= 482.5) |
| Theta (z'- axis) | (x= 0, y= 0.9137, z= -1) |
| Phi (x'- axis) | (x= 1, y= 0, z= 0) |
| Frequency | 2.40477 ... 2.40477 |

**Fig. 7.8** The parabolic reflector antenna illuminated with farfield source.



farfield (f=2.4048) [ffs2]

| | |
|---|---|
| Type | Farfield |
| Approximation | enabled (kR >> 1) |
| Component | Abs |
| Output | Directivity |
| Frequency | 2.40477 GHz |
| Rad. Effic. | -0.02597 dB |
| Tot. Effic. | -0.6496 dB |
| Dir. | 23.31 dBi |

**Fig. 7.9** 3D radiation pattern of final antenna assembly. The reflector is illuminated with helical antenna with 7 turns.

Step 5

Finally, the antenna efficiency was calculated and plotted in Fig. 7.10.

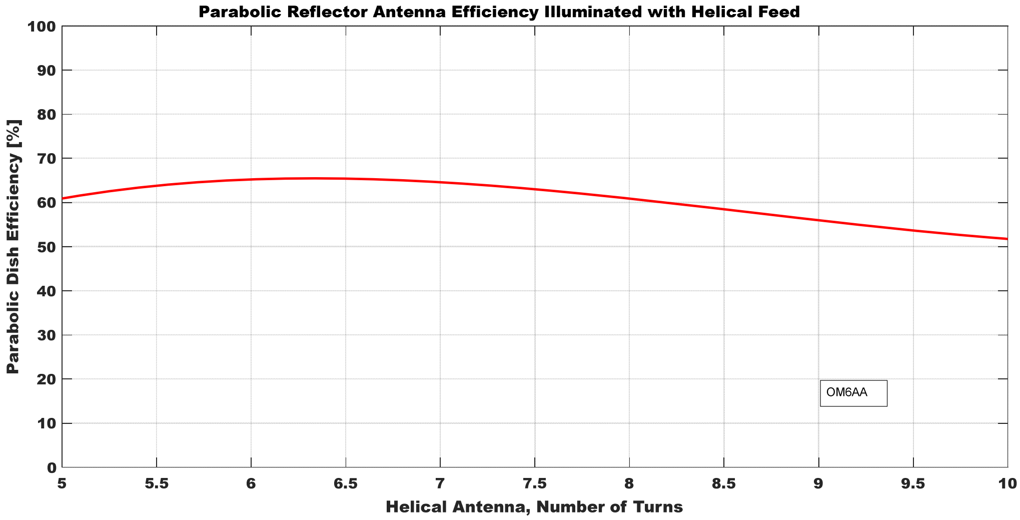


**Fig. 7.10** Efficiency of the Gibertini, model OP 75 LN, reflector antenna illuminated with the helical feed.

Using the real radiation pattern of the helical feed for antenna analysis, we found, that optimal helical antenna for given parabolic reflector is with **5.5 up to 7.5 turns**. The efficiency curve is very shalow, so number of turns is not critical. The difference in antenna performance within range 5 to 8 turns is only 0.3 dB, what can be less than power loss on a bad antenna connector or antenna match with VSWR =1.7.

## Note

In the analysis method used in Example 3, we split the solution on two parts. First one was a feed analysis and second one was a reflector analysis. This helped us to deal with a reasonable amount of software meshcells on which the calculation time and computer requirements depend. However due to the feed size, the reflector works on the edge of near-field region. This was probably the cause in the deviations of the phase center positions.

I was wondering how credible this method is. So, I constructed the whole antenna model in T-solver, see Fig 7.11 to validate calculated data. More than 11 million meshcells were used for calculation. The difference between methodes was 0.5 dB, which is very good agreement. The differnce can be addressed to phase deviations or even solver settings.

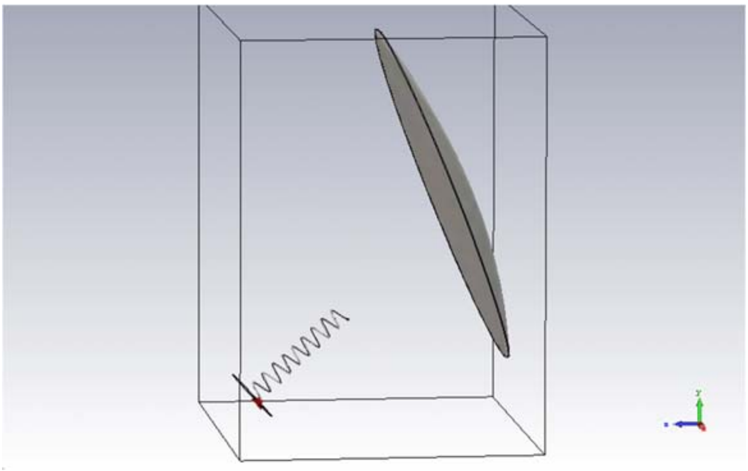


**Fig. 7.11** Model of parabolic reflector fitted with 9 turns helical feed

## Conclusion

The QO100 satellite, besides of its main communication function, is an excellent tool for the RF antenna experimenters. Using the satellite transponder allows them to easily measure and test developed antennas in real world conditions. Neither an anechoic chamber nor expensive RF measuring instruments are required. The transponder is very sensitive and it forgives different less successful design deficiencies. Ultimately, it will allow you to gain valuable knowledge about the RF engineering.

## References

[1] Milligan, T. A. *“Modern Antenna Design“*. Wiley-IEEE Press, 2nd edition, July 11, 2005.

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